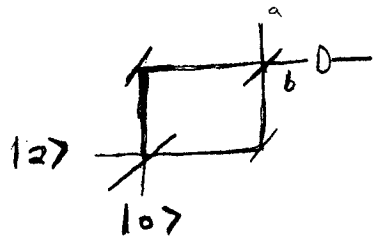
Homework #2



i) Interferometer for $|2,0\rangle$

From last time:

$U_{B.S.}\,|2,0\rangle = \frac{1}{2}[|2,0\rangle - |0,2\rangle] + \frac{i}{\sqrt{2}}|1,1\rangle$ // $U_{B.S.}\,|0,2\rangle = \frac{1}{2}[|0,2\rangle - |2,0\rangle] + \frac{i}{\sqrt{2}}|1,1\rangle$

$U_{B.S.}\,|1,1\rangle = \frac{i}{\sqrt{2}}[|2,0\rangle + |0,2\rangle]$

$U_{F.S.} = e^{i\omega a l a^\dagger a/c}$

$|\psi_f\rangle = U_{B.S.}\,U_{F.S.}\,U_{B.S}\,|2,0\rangle$

$= U_{B.S.}\,U_{F.S.}\left[\frac{1}{2}[|2,0\rangle - |0,2\rangle] + \frac{i}{\sqrt{2}}|1,1\rangle\right]$

$e^{i\phi a^\dagger a}\,|n,m\rangle = \left(1 + i\phi a^\dagger a - \frac{\phi^2 (a^\dagger a)^2}{2!} - \dots\right)|n,m\rangle$

$= \left(1 + i\phi n - \frac{\phi^2 n^2}{2!} - \dots\right)|n,m\rangle = e^{i\phi n}\,|n,m\rangle$

Fmid:
$\frac{1}{4}[e^{i\phi}+1][e^{-i\phi}+1]$
$= \frac{1}{2}[\cos\phi + 1]$
$= \cos\left(\frac{\phi}{2}\right)^2$

$[e^{i2\phi}+1]e^{-i\phi} + [e^{-i2\phi}+1]e^{i\phi}$
$= 4\cos\phi$

$\therefore |\psi_f\rangle = U_{B.S}\left[\frac{1}{2}\left[e^{i2\omega a l/c}\,|2,0\rangle - |0,2\rangle\right] + \frac{i}{\sqrt{2}}e^{i\omega a l/c}\,|1,1\rangle\right]$

$= \frac{1}{2}\left[e^{i2\omega a l/c}\right]\left[\frac{1}{2}[|2,0\rangle - |0,2\rangle] + \frac{i}{\sqrt{2}}|1,1\rangle\right]$

$-\frac{1}{2}\left[\frac{1}{2}[|0,2\rangle - |2,0\rangle] + \frac{i}{\sqrt{2}}|1,1\rangle\right] - \frac{1}{2}e^{i\omega a l/c}[|2,0\rangle + |0,2\rangle]$

$b^\dagger b\,|\psi_f\rangle = -\frac{1}{2}e^{i2\omega a l/c}\left[|0,2\rangle - \frac{i}{\sqrt{2}}|1,1\rangle\right] + \frac{1}{2}|0,2\rangle + \frac{i}{\sqrt{2}}|1,1\rangle - e^{i\omega a l/c}|0,2\rangle$

$\langle\psi_f|b^\dagger b|\psi_f\rangle = \frac{1}{2}\left[e^{-i2\omega a l/c}+1\right]\left[\frac{1}{2}[\langle 2,0| + \langle 0,2|] - \frac{i}{\sqrt{2}}\langle 1,1|\right]$

$-\frac{1}{2}\left[e^{-i\omega a l/c}[\langle 2,0| + \langle 0,2|]\right]\,|\,b^\dagger b\,|\psi_f\rangle$

$= \frac{1}{2}\left[e^{i2\omega a l/c}+1\right]\cdot\frac{1}{4}\left[e^{-i2\omega a l/c}+1\right] - \frac{1}{4}\left[e^{i2\omega a l/c}+1\right]e^{-i\omega a l/c}$

$- e^{i\omega a l/c}\cdot\frac{1}{4}\left[e^{-i2\omega a l/c}+1\right] + \frac{1}{2} + \frac{1}{8}\left[e^{i2\omega a l/c}+1\right]\left[e^{-i2\omega a l/c}+1\right]$

$= \frac{1}{2}[1+\cos 2\omega a l/c] + \frac{1}{2} - \cos\omega a l/c$ $\quad b^\dagger b = 1+\cos 2\omega a l/c$

$= [\cos\omega a l/c]^2$