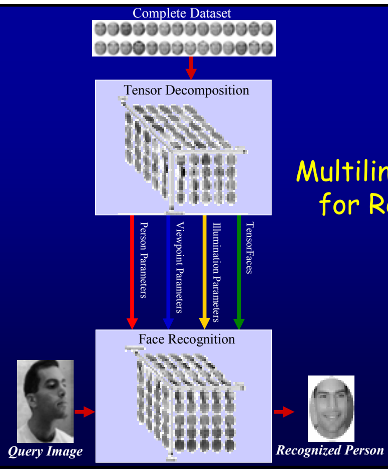
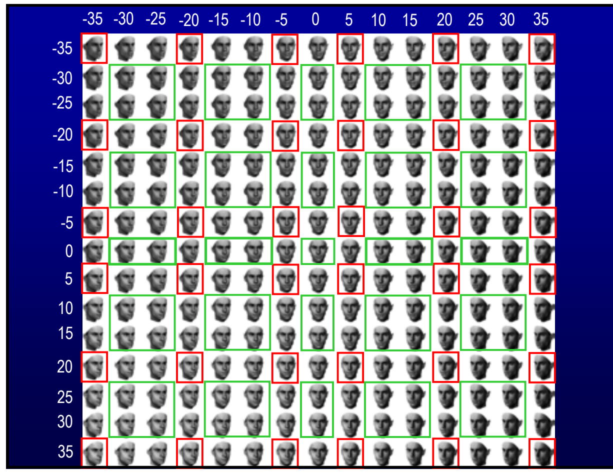


Multilinear Projection for Recognition



Results

[Vasilescu & Terzopoulos, CVPR 2005]

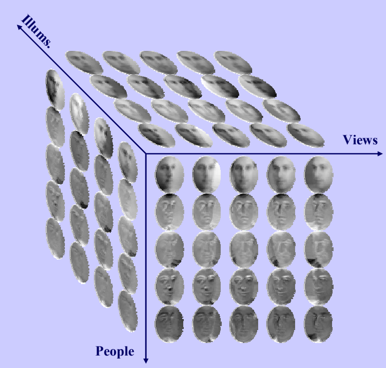
- Data Set - 16,875 images
  - 75 people
  - 15 viewpoints
  - 15 illuminations

Training Images - 2,700
- 75 people
- 6 viewpoints
- 6 illuminations

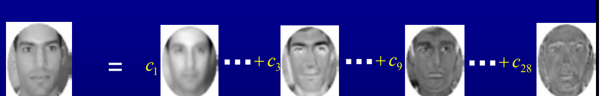
Test Images:
- 75 people
- 9 viewpoints
- 9 illums

Linear Models		Multilinear Models	
PCA	ICA	TensorFaces (MPCA)	Independent TensorFaces (MICA)
83%	89%	93%	97%

TensorFaces Recognition



Linear Representation:



$$\mathbf{d}_i = \mathbf{U}\mathbf{c}_i$$

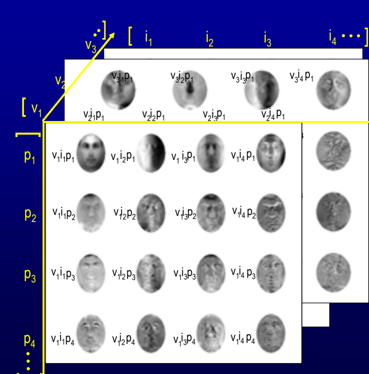
$$\mathbf{U}^T\mathbf{d}_i = \mathbf{c}_i$$

Unknown coefficient vector

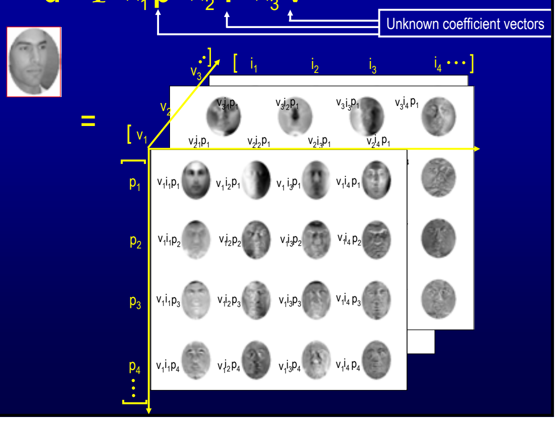
Projection Operator

Multilinear Representation:

=



$$\vec{d} = \mathcal{T} \times_1 \vec{p}^T \times_2 \vec{i}^T \times_3 \vec{v}^T$$



Inverse Tensor

- **Definition:**
  - *If $\mathcal{I}$ is an identity tensor, then $\mathcal{B}$ is called a mode-n inverse of tensor $\mathcal{A}$ if and only if $\mathcal{A} \times_n \mathcal{B} = \mathcal{I}$ and $\mathcal{B} \times_n \mathcal{A} = \mathcal{I}$*
  - *Mode-n Inverse tensor of $\mathcal{A}$ is denoted as $\mathcal{A}^{-n}$*
- **Definition,** Mode-n Product extension:

  $\mathcal{A} \times_n \mathcal{B} \Longleftrightarrow \mathbf{B}_{(n)} \mathbf{A}_{(n)}$

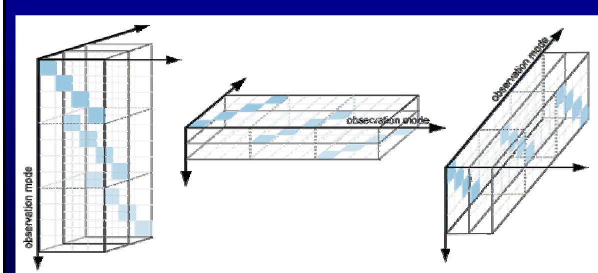
Mode-n Identity Tensor

- **Definition 8 (Identity Tensor)** *The mode-n identity*
- *tensor satisfies the following criteria:*

1. *$\mathcal{I}$ is a mode-n identity tensor if and only if $\mathcal{I} \times_n \mathcal{A} = \mathcal{A}$.*
2. *$\mathcal{I}$ is a multiplicative identity.*
3. *The mode-n identity tensor $\mathcal{I} \in \mathbb{R}^{I_1 \times I_2 \cdots \times J_n \cdots I_N}$ where*

- *$J_n = I_{n+1} \ldots I_N I_1 I_2 \ldots I_{n-1}$ is a tensorized identity matrix c*
- *dimensionality $J_n \times (I_{n+1} \ldots I_N I_1 \ldots I_{n-1})$.*

Mode-n Identity Tensor



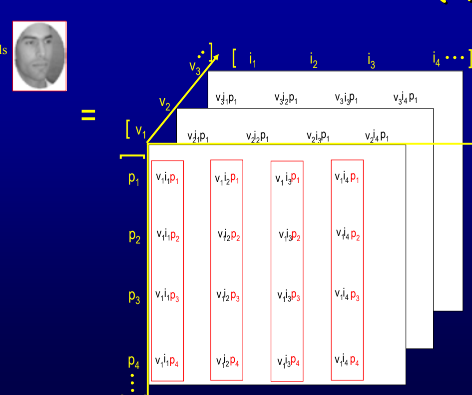
Mode-n Pseudo Inverse

- **Definition 9 (Pseudo-Inverse Tensor)** *The mode-n*
- *pseudo-inverse tensor, $\mathcal{A}^{+n}$, of tensor $\mathcal{A}$ satisfies th*
- *following criteria:*

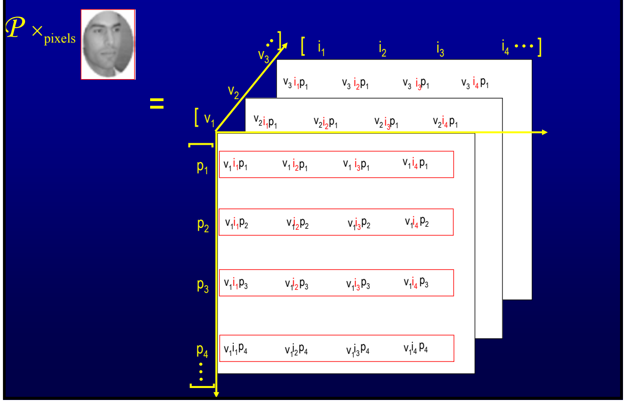
1. $((\mathcal{A} \times_n \mathcal{A}^{+n}) \times_n \mathcal{A}) = \mathcal{A}$
2. $((\mathcal{A}^{+n} \times_n \mathcal{A}) \times_n \mathcal{A}^{+n}) = \mathcal{A}^{+n}$
3. The mode-n pseudo-inverse tensor $\mathcal{A}^{+n}$ of an N^{th} order tensor $\mathcal{A} \in$ IRI1×I2×⋯×IN is the tensorized pseudo-inverse of $\mathbf{A}_{(n)}$; i.e., the mode-n flattened version of $\mathcal{A}^{+n}$ is $\mathbf{A}^{+}_{(n)}$

Contribution Tensor – Rank (1,…,1)

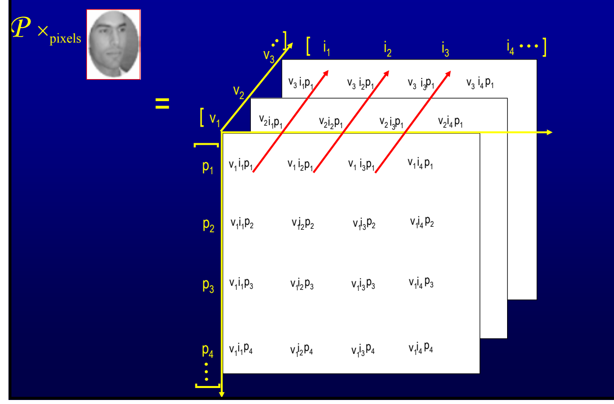
$\mathcal{P} \times_{\text{pixels}}$



Contribution Tensor – Rank (1,…,1)



Contribution Tensor – Rank (1,…,1)



Contribution Tensor – Rank-(1,1,1)

$$\mathcal{P} \times_{pixels} \mathbf{d}^T \approx \mathbf{p} \circ \mathbf{v} \circ \mathbf{l}$$

Projection Operator

Response Tensor

where $\mathbf{P}_{(pixels)} = \mathbf{T}^{+T}_{(pixels)}$

Multilinear Projection

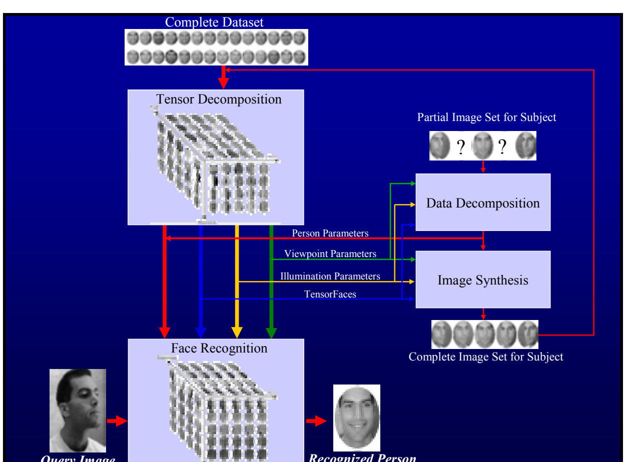
1. Compute the Projection Tensor:

$$\mathbf{P}_{(\text{observ. mode})} = \mathbf{T}^{+T}_{(\text{observ. mode})}$$

2. Compute the Response Tensor:

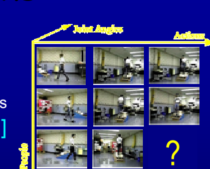
$$\begin{aligned} \mathcal{R} &= \mathcal{P} \times_{\text{observ. mode}} \mathbf{d}^T \\ &= \mathcal{J} \times_{\text{observ. mode}} (\mathbf{l}^T \otimes \mathbf{v}^T \otimes \mathbf{p}^T) \\ &= (\mathbf{p} \circ \mathbf{v} \circ \mathbf{l}) \end{aligned}$$

3. Extract the coefficient vectors by factorizing the Response Tensor by using N-mode SVD.



Other Applications

- Human Motion Signatures
  - 3-Mode Decomposition, Recognition, & Synthesis
- [Vasilescu ICPR 02, CVPR 01, SIGGRAPH 01]



- Image-Based Rendering
  [Vasilescu & Terzopoulos, SIGGRAPH 04]